Control of Low-Speed Airfoil Aerodynamics

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Nomenclature

C_D = drag coefficient ($\equiv 2D/\rho U_\infty^2 c$)
C_f = local skin-friction coefficient ($\equiv 2\tau_w/\rho U_0^2$)
C_L = lift coefficient ($\equiv 2L/\rho U_\infty^2 c$)
C_q = suction coefficient ($\equiv |v_w|/U_0$)
c = airfoil's chord
D = drag force per unit span
L = lift force per unit span
P = instantaneous hydrostatic pressure
P_0 = pressure outside boundary layer
$\bar{P}$ = mean pressure
R = wall's radius of curvature
R_{δ^*} = displacement thickness Reynolds number ($\equiv U_0\delta^*/\nu$)
R_θ = momentum thickness Reynolds number ($\equiv U_0\delta_\theta/\nu$)
Re = Reynolds number based on distance from leading edge (or chord) and freestream velocity
T = instantaneous temperature
$\bar{T}$ = mean temperature
U_i = instantaneous velocity component
$\bar{U}_i$ = mean velocity component
U_0 = velocity outside the boundary layer
U_∞ = freestream velocity
u_i = fluctuating velocity component
u^* = friction velocity ($\equiv \sqrt{\tau_w/\rho}$)
v_w = normal velocity of fluid injected or withdrawn through the wall
x_i = Cartesian coordinates
x = streamwise distance from leading edge
y = normal distance from the wall
y^+ = normal distance in wall units ($\equiv yu^*/\nu$)
z = spanwise coordinate
α = angle of attack
δ = boundary-layer thickness
δ_θ = momentum thickness
δ^* = displacement thickness
μ = dynamic coefficient of viscosity
ν = kinematic viscosity
ν/u^* = viscous length scale (wall unit)
ρ = density
$-\rho\overline{uv}$ = tangential Reynolds stress
τ_w = shear stress at the wall ($= \rho u^{*2}$)
$[\Omega_z]_0$ = instantaneous spanwise vorticity at the wall ($\equiv -[\partial U/\partial y]_0$)
$[\bar{\Omega}_z]_0$ = mean spanwise vorticity at the wall ($\equiv -[\partial \bar{U}/\partial y]_0$)

I. Introduction

INSECTS, birds, and bats have perfected the art of flight through millions of years of evolution. Man's dream of flying dates back to the early Greek myth of Daedalus and his son Icarus, but the first successful heavier-than-air flight took place a mere 86 yr ago. Today, the Reynolds numbers for natural and man-made fliers span the amazing range from 10^2 to 10^9; insects are at the low end of this spectrum, and huge airships occupy the high end.[1]

The function of the airfoil section on those fliers is to produce lift. Inevitably, viscous effects, compressibility effects, and the finite span of the lifting surface all ensure that drag is also produced. A thrust must be generated by some sort of a power plant to overcome this streamwise resistance to the motion. The lift-to-drag ratio is a measure of the effectiveness of the airfoil. In general, this ratio is very low at low Reynolds numbers and improves with increases in this parameter. As shown in Fig. 1, taken from Ref. 2, the maximum $[C_L/C_D]$ improves dramatically in the range of Reynolds numbers of 10^4–10^6. Below 10^4, typical of insects and small model airplanes, the boundary layer around the lifting surface is laminar. Stalling in this case is caused by an abrupt separation of the laminar flow near the leading edge as the angle of attack is increased to modest values. The maximum lift is limited, and the drag increases significantly when the lifting surface stalls.

For $Re > 10^6$, typical of large aircraft, boundary-layer transition to turbulence usually takes place ahead of the theoretical laminar separation point. A turbulent boundary layer can

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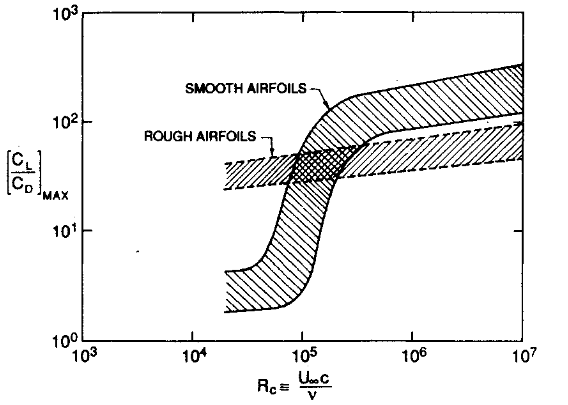


Fig. 1 Airfoil performance as a function of chord Reynolds number.[2]

negotiate quite severe adverse pressure gradients without separation, and this kind of lifting surface often experiences a trailing-edge stall at relatively high angles of attack. The stall is preceded by a movement of the separation point forward from the trailing edge with increasing incidence.[3]

In the range of Reynolds numbers of 10^4–10^6, termed low Reynolds number for the purpose of this article,[4] many complicated phenomena take place within the boundary layer. Separation, transition, and reattachment could all occur within a short distance and affect the performance of the lifting surface. The laminar separation bubble that commonly forms in this range of Reynolds numbers plays an important role in determining the boundary-layer behavior and the stalling characteristics of the airfoil.[5] As indicated in Figure 1, the maximum lift-to-drag ratio for a smooth airfoil increases by two orders of magnitude in this Reynolds number regime.

There are a variety of passive and active techniques to effect a beneficial change in the complex flowfield that characterizes this intermediate range of Reynolds numbers. Roughness and shaping are among the simplest passive methods to ensure flow attachment beyond a critical angle of attack and, thus, an improved performance. Wall transpiration or heat transfer are examples of active control methods to improve the lift-to-drag ratio.

The object of this paper is to survey available and contemplated flow-control methods particularly suited for low-Reynolds-number airfoils. Although some of these techniques were originally developed for military or large transport aircrafts operating at much higher Reynolds numbers, they can be adopted to the low-speed situation. Section II gives a brief overview of the fluid dynamics of these lifting surfaces. The different control goals and their interrelations are summarized in Sec. III. The governing equations needed to present a unified view to explain the different control methods are recalled in Sec. IV. Sections V through VII detail the methods for separation/reattachment, transition, and drag control, respectively. Finally, a brief summary is given in Sec. VIII.

II. Low-Reynolds-Number Airfoils

In the range of Reynolds numbers of 10^4–10^6, a substantial improvement in the lift-to-drag ratio of an airfoil takes place. According to Carmichael,[1] this is the Reynolds number regime where we find man and nature together in flight; large soaring birds, large radio-controlled model aircraft, foot-launched ultralight, man-carrying hang gliders, human-powered aircraft, and remotely piloted vehicles (RPVs) used for military and scientific sampling, monitoring, and surveillance. Three review articles of low-Reynolds-number aerodynamics are particularly recommended.[4-6]

In this range of Reynolds numbers, very complex flow phenomena take place within a short distance on the upper surface of an airfoil at incidence. Unless artificially tripped, the boundary layer remains laminar at the onset of pressure recovery, and the airfoil's performance is then entirely dictated by the laminar flow's poor resistance to separation. The separated flow forms a free-shear layer which is highly unstable, and transition to turbulence is readily realized. Subsequent reattachment of the separated region may take place because of the increased entrainment associated with the turbulent flow. Provided that the high-speed fluid entrained into the wall region supplies sufficient energy to maintain the circulating motion against dissipation, a separation bubble forms.

The precise conditions for the occurrence of separation, transition, and reattachment, in other words for the formation of a laminar separation bubble, depend on the Reynolds number, the pressure distribution, the surface curvature, the surface roughness, and the freestream turbulence, as well as other environmental factors. If the Reynolds number is sufficiently high, transition takes place near the minimum pressure point ahead of the location at which separation would have occurred if the boundary layer had remained laminar. For moderate Reynolds numbers, separation takes place prior to transition. The laminar boundary layer can only support very small adverse pressure gradient without separation. In fact, if the ambient incompressible fluid decelerates in the streamwise direction faster than $U_0 \sim x^{-0.09}$, the flow separates.[7] The separated flow will not reattach to the surface, and no bubble will be formed if the Reynolds number is sufficiently low. However, for the intermediate Reynolds number range (typically 10^4–10^6), the separated flow proceeds along the direction of the tangent to the surface at the separation point,[8] and transition to turbulence takes place in the free-shear layer due to its increased transition susceptibility. Subsequent turbulent entrainment of high-speed fluid causes the flow to return to the surface, thus, forming what is known as a laminar separation bubble, as sketched in Fig. 2. Downstream of the point of reattachment, the newly formed turbulent boundary layer is capable of negotiating quite severe adverse pressure gradients without separation. The ability of a turbulent boundary layer to resist separation improves as the Reynolds number increases.[4]

It is clear from the above arguments that bubble formation is confined to a certain range of Reynolds numbers and that this range changes from one airfoil to another as well as from one environment to another. A rough rule according to Carmichael[1] is that the Reynolds number based on freestream velocity and the distance from separation to reattachment is approximately 5×10^4. In general, then, an airfoil with chord Reynolds number less than 5×10^4 will experience laminar separation with no subsequent reattachment. For chord Reynolds numbers slightly higher than 5×10^4, a long bubble is expected. Shorter bubbles are formed at higher Reynolds numbers. Tani[5] asserts that a Reynolds number typical of local conditions in the boundary layer is more appropriate to characterize a separation bubble than the chord Reynolds number. Typically, the Reynolds number, based on the boundary layer's displacement thickness and the velocity just outside the rotational flow region at the point of separation, is more than 500 for a short bubble and less than 500 for a long one. The corresponding bubble's streamwise extent, normalized with

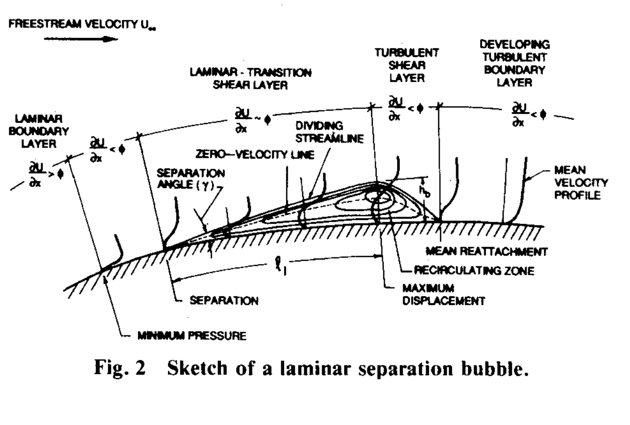


Fig. 2 Sketch of a laminar separation bubble.

the displacement thickness at the point of separation, is 10^2 and 10^4, respectively.[5] Note that the bubble gets shorter at higher R_{δ^*}; and at $R_{\delta^*} \simeq 6000$, bubble formation is precluded (bubble's streamwise extent approaches zero) by transition to turbulence in the boundary layer. The displacement thickness δ^* for a laminar boundary layer near the leading edge of an airfoil is extremely small and cannot be accurately measured. Instead the momentum thickness δ_θ is computed from the pressure distribution using Thwaites' formula,[9] and the ratio δ^*/δ_θ at the separation point is assumed to be 3.7.

The short separation bubble generally has a length of the order of a few percent of the chord. It merely represents a transition-forcing (tripping) mechanism to allow reattachment of an otherwise separated shear layer. Such a bubble does not greatly affect the peak suction as determined from the potential flow solution around the airfoil. Except for the appearance of a minute bump in the lift curve (C_L vs α curve), the presence of a short bubble has no significant effect on the pressure distribution around the lifting surface, as depicted in Fig. 3a. On the other hand, a long bubble may be as much as 0.2c–0.3c and significantly changes the pressure distribution by effectively altering the shape over which the outer potential flow is developed. In this case, the sharp suction peak near the leading edge is generally not realized, and a suction plateau of a reduced level extends over the region occupied by the bubble (see Fig. 3b). A long bubble tends to increase in length as

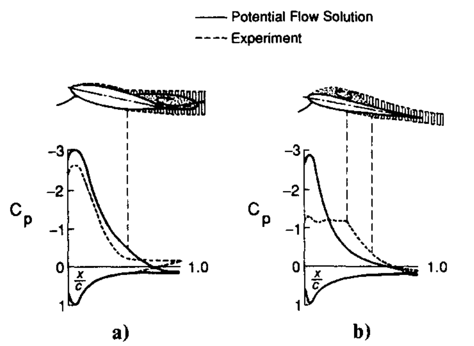


Fig. 3 Qualitative pressure distributions for two airfoils at incidence; a) Short bubble on upper surface and subsequent rear separation of turbulent boundary layer; b) Long bubble.

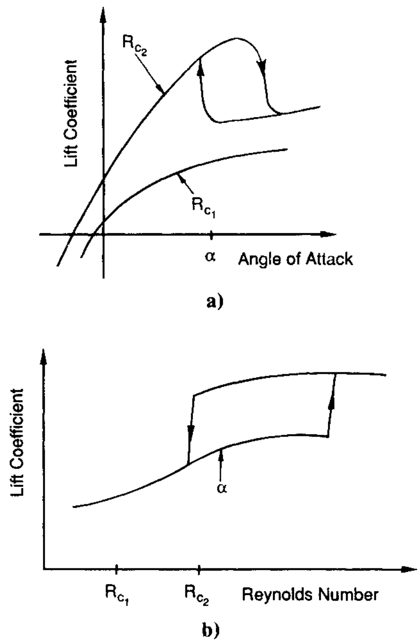


Fig. 4 Illustration of lift hysteresis as the angle of attack or the Reynolds number is recycled.

incidence is increased leading to a corresponding decrease in the slope of the lift curve as well as an increase in the pressure drag.

In general, the lift-to-drag ratio is higher for an airfoil having a shorter bubble. Depending on many factors, a short bubble forming at low incidence may move forward and contract in streamwise extent as the angle of attack is increased.[5] Within the bubble, a small region of constant pressure exists followed by pressure recovery. At higher incidence, the bubble bursts and no longer reattaches thus ensuing a leading-edge stall.[10] This process is often irreversible, meaning that reducing the angle of attack will not immediately unburst the bubble. As sketched in Fig. 4, strong lift hysteresis effects are thus observed as the attack angle or the Reynolds number is recycled.[1,11-14] Similar hysteresis effects are also seen in drag.

For thin airfoils of small nose radius, pressure recovery commences very near the leading edge, and the adverse pressure gradients are severe at high angles of attack. A separation bubble may occur on these airfoils even at chord Reynolds numbers exceeding 10^6. At large incidence, the short bubble breaks down into a long one. With increasing angles of attack, the reattachment point moves progressively backward until it reaches the trailing edge, at which stage its maximum thickness is typically 3% of the chord. A further increase in incidence leads to completely detached flow and the so-called thin airfoil stall. A comprehensive review of the different kinds of stall on thin airfoils is given by Crabtree.[15]

Aerodynamics data for both thin and thick airfoils in the low-Reynolds-number regime are accessible in several recent papers and conference proceedings.[16-25] Available experimental data on bubble's formation and bursting indicates that transition to turbulence in the separated shear-layer and subsequent reattachment will occur if the Reynolds number based on displacement thickness at the point of laminar separation exceeds a critical value that is not necessarily universal (Tani-Owen-Klanfer criterion). A lower limit for this Reynolds number seems to be $R_{\delta^*} \simeq 350$. Bursting occurs if the pressure recovered in the reattachment process in terms of the dynamic pressure at separation (pressure recovery coefficient) exceeds a certain critical value (Crabtree criterion). Again, this critical value changes from one airfoil to another, but an upper limit of 0.35 appears to be valid for many shapes. Crabtree[26] assumes that bubble's breakdown occurs because there exists a maximum possible value of pressure that can be recovered in the turbulent entrainment process that causes the flow reattachment. This implies the existence of a maximum possible value of the shear stress set up in the turbulent entrainment region so as to counteract the pressure gradient. At breakdown, caused by either an increase in incidence or a decrease in Reynolds number, the Tani-Owen-Klanfer criterion is satisfied, but the Crabtree criterion is about to be violated.

The question of primary concern to us in this article is how to control the flow around a low-Reynolds-number airfoil to achieve an improved performance. The interrelation among the different control goals is particularly salient when a separation bubble exists, and this issue will be tackled in the next section.

III. Control Goals and Their Interrelations

According to Tani,[5] all three kinds of stall, trailing-edge stall, leading-edge stall, and thin-airfoil stall, may occur for a given airfoil at different Reynolds numbers or for different airfoils at a given Reynolds number. A particular lifting surface produces higher lift at higher incidence, limited by the angle at which the airfoil stalls. At that point, drag increases drastically, and the lifting-surface performance deteriorates rapidly. Flow control is aimed at improving this performance. Among the practical considerations that must be considered for both active and passive control devices are their cost of construction and operation, complexity, and potential trade-offs or penalties associated with their use. It is this latter point in particular that presents an additional degree of complexity for controlling low-Reynolds-number lifting surfaces. Achieving a beneficial effect for one control goal may very well adversely affect another goal, and design compromises must often be made.

Among the desired goals of external flow modification are separation/reattachment control, lift enhancement, transition delay/advancement, and drag reduction. These objectives are not necessarily mutually exclusive, and for low-Reynolds-number lifting surfaces, the interrelations among these goals are particularly conspicuous presenting an additional degree of complexity. As mentioned before, in the range of Reynolds numbers of 10^4–10^6, a laminar separation bubble may form and may have a dominant effect on the flowfield and the airfoil's performance. Figure 5 is a schematic representation of the interrelation between one control goal and another. If the boundary layer becomes turbulent, its resistance to separation is enhanced, and more lift could be obtained at increased incidence. On the other hand, the skin-friction drag for a laminar boundary layer can be as much as an order of magnitude less than that for a turbulent one. If transition is delayed, lower skin friction is achieved. However, the laminar boundary layer can only support very small adverse pressure gradient without separation, and subsequent loss of lift and increase in form drag occur. A free-shear layer forms downstream of the laminar separation point and for moderate Reynolds number transition to turbulence takes place along this shear layer. Increased entrainment of high-speed fluid due to the turbulent mixing may result in reattachment of the separated region and formation of a laminar separation bubble. At higher incidence, the bubble breaks down either separating completely or forming a longer bubble. In either case, the form drag increases, and the lift-curve's slope decreases. The ultimate goal of all this is to improve the airfoil's performance by increasing the lift-to-drag ratio. However, induced drag is caused by the lift generated on a lifting surface with a finite span. Moreover, more lift is generated at higher incidence but form drag also increases at these angles.

All of the above point to potential conflicts as one tries to achieve a particular control goal only to adversely affect another goal. An ideal method of control that is simple, inexpensive to build and operate, and does not have any tradeoffs does not exist, so there must be compromises to achieve a particular design goal. Keeping this in mind, we now proceed to review the control methods available for a low-speed lifting surface. A brief review of the governing equations will be useful in presenting a unified view, based on vorticity considerations, of the different control methods to achieve a variety of end results.

IV. Governing Equations

To examine the local and instantaneous effects of the different control tools, consider a rigid surface such that $U = W = 0$ everywhere along it. At the surface, $y = 0$, the Navier-Stokes equations for an incompressible fluid read

$$v_w\left[\frac{\partial U}{\partial y}\right]_0 + \frac{1}{\rho}\left[\frac{\partial P}{\partial x}\right]_0 - \left[\frac{\partial \nu}{\partial y}\right]_0\left[\frac{\partial U}{\partial y}\right]_0 = \left[\nu\frac{\partial^2 U}{\partial y^2}\right]_0 \tag{1}$$

$$0 + \frac{1}{\rho}\left[\frac{\partial P}{\partial y}\right]_0 - 0 = \left[\nu\frac{\partial^2 V}{\partial y^2}\right]_0 \tag{2}$$

$$v_w\left[\frac{\partial W}{\partial y}\right]_0 + \frac{1}{\rho}\left[\frac{\partial P}{\partial z}\right]_0 - \left[\frac{\partial \nu}{\partial y}\right]_0\left[\frac{\partial W}{\partial y}\right]_0 = \left[\nu\frac{\partial^2 W}{\partial y^2}\right]_0 \tag{3}$$

where the subscript $[\ ]_0$ indicates flow quantities computed at the surface; U, V, and W are the instantaneous velocity components in the x, y, and z directions, respectively; v_w is the normal velocity at the wall (positive for injection and negative for suction); P is the instantaneous pressure; and ν is the kinematic viscosity, which in general varies with space and

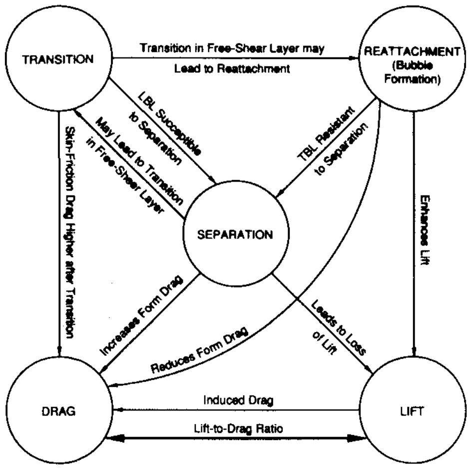


Fig. 5 Inter-relation between flow control goals.

time as a result of surface heating/cooling, film boiling, cavitation, sublimation, chemical reaction, wall injection of higher/lower viscosity fluid, or the presence of shear thinning/thickening additive.

The above equations indicate that the instantaneous flux of spanwise vorticity ($\partial U/\partial y$) and streamwise vorticity ($\partial W/\partial y$) at the wall (Assuming spatially uniform v_w, $\partial V/\partial x$, and $\partial V/\partial z$ are both zero at the surface. Moreover, normal vorticity must vanish at $y = 0$ due to the no-slip condition.) could be affected by suction/injection, streamwise or spanwise pressure gradient, or change of viscosity in the normal direction. Analogies between these stimuli are often made based on Eqs. (1-3) or the corresponding equation for the mean streamwise momentum

$$v_w\left[\frac{\partial \bar{U}}{\partial y}\right]_0 + \frac{1}{\rho}\frac{\mathrm{d}P_0}{\mathrm{d}x} - \frac{\mathrm{d}\nu}{\mathrm{d}\bar{T}}\left[\frac{\partial \bar{T}}{\partial y}\frac{\partial \bar{U}}{\partial y}\right]_0 + \left[\frac{\partial \overline{uv}}{\partial y}\right]_0 = \left[\nu\frac{\partial^2 \bar{U}}{\partial y^2}\right]_0 \tag{4}$$

where $\bar{T}$ is the mean temperature field, and $\bar{U}$ is the time-averaged streamwise velocity. In this equation, viscosity is assumed to change as a result of surface heat transfer. The right-hand side of Eq. (4) is the flux of mean spanwise vorticity $\bar{\Omega}_z = -\partial\bar{U}/\partial y$ at the surface. In the absence of suction/injection, pressure gradient, and surface heating/cooling, the first three terms on the left-hand side of Eq. (4) vanish.

For a laminar boundary layer, the Reynolds stress term vanishes. For the turbulent case, the fourth term on the left-hand side is the slope of the normal profile of $\overline{uv}$ at $y = 0$. This term could be asymptotically estimated as the wall is approached. Consider a Taylor's series expansion in powers of y in the neighborhood of the point $y = 0$. As a result of the no-slip condition, the streamwise velocity fluctuations u varies at least as y. To conserve mass, the normal velocity fluctuations must vary as y^2. It follows then that very near the wall (within the viscous sublayer), the tangential Reynolds stress $\overline{uv}$ varies at least as y^3 and that $\partial\overline{uv}/\partial y$ varies as y^2. At the wall itself, $y = 0$ and $[\partial\overline{uv}/\partial y]_0 = 0$; although close to the wall, the slope of the tangential Reynolds stress profile is quite large.

The above arguments together with Eq. (4) indicate that the mean streamwise velocity profile for the canonical turbulent boundary layer (two-dimensional, incompressible, isothermal, zero pressure gradient over an impervious, rigid surface) will have a zero curvature at the wall. Notwithstanding this common characteristic with the Blasius boundary layer, the turbulent boundary layer is quite different from the laminar one. As pointed out by Lighthill,[27] the turbulent mixing concentrates most of the mean vorticity much closer to the wall as compared to the laminar case. The mean vorticity at the wall $[\partial\bar{U}/\partial y]_0$ is typically an order of magnitude larger than that in the laminar case. The turbulent mixing also causes the mean vorticity to migrate away from the wall, and about 5% of the total is found much farther from the surface. The flux of mean spanwise vorticity is zero at the wall itself but very large close to it reaching a maximum at about the same location where the root-mean-square vorticity fluctuations peaks (near the edge of the viscous sublayer). This trait is responsible for the turbulent boundary layer's resistance to separation.

For a steady, incompressible flow around a two-dimensional or axisymmetric surface of small curvature, the time-averaged continuity and streamwise momentum equations can be integrated in the normal direction to yield the von Karman integral equation

$$C_f = 2\,\frac{d\delta_\theta}{dx} + 2\delta_\theta\left[\left(2+\frac{\delta^*}{\delta_\theta}\right)\frac{1}{U_0}\frac{dU_0}{dx} + \frac{1}{R}\frac{dR}{dx}\right] - 2\,\frac{v_w}{U_0} \qquad (5)$$

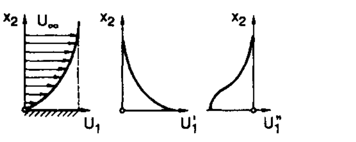
where C_f is the local skin-friction coefficient, δ^* and δ_θ are the displacement and momentum thicknesses, respectively, U_0 is the velocity outside the boundary layer, R is the radius of curvature of the wall, and v_w is the normal velocity of fluid injected through the surface. Eq. (5) is valid for both laminar and turbulent boundary layers.

In the next three sections, available and contemplated flow control methods for low-Reynolds-number lifting surfaces are discussed. The equations developed in this section for laminar and turbulent boundary layers will help in presenting a unified view of the different control techniques. A recent comprehensive review of boundary layer control is available.[28]

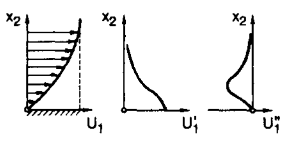
V. Separation/Reattachment Control

Fluid particles in a boundary layer are slowed down by wall friction. If the external potential flow is sufficiently retarded, for example due to the presence of an adverse pressure gradient, the momentum of those particles will be consumed by both the wall shear and the pressure gradient. At some point (or line), the viscous layer departs or breaks away from the bounding surface. The surface streamline nearest to the wall leaves the body at this point, and the boundary layer is said to separate.[29] At separation, the rotational flow region next to the wall abruptly thickens, the normal velocity component increases, and the boundary-layer approximations are no longer valid. Due to the large energy losses associated with boundary-layer separation, the performance of a lifting surface is often controlled by the separation location. If separation is postponed, the pressure drag is decreased, and the circulation and hence the lift at high angles of attack is enhanced.

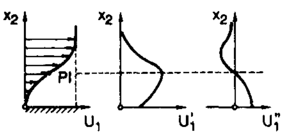
Prandtl[30] first explained the mechanics of separation. He provided a precise criterion for its onset for the case of a steady, two-dimensional boundary layer developing over a fixed wall. In case such a flow is retarded, the near-wall fluid may have insufficient momentum to continue its motion and is brought to rest at the point of separation. Fluid particles behind this point move in a direction opposite to the external stream, and the original boundary-layer fluid passes over a region of recirculating flow. Since the velocity at the wall is always zero, the gradient $[\partial U/\partial y]_0$ will be positive upstream of separation, zero at the point of separation, and negative in the reverse flow region. (Mean velocity $\overline{U}$ is used in the derivative for the case of a turbulent flow.) The velocity profile at separation must then have a positive curvature at the wall. However, $[\partial^2 U/\partial y^2]$ is negative at a large distance from the wall, which means the velocity profile at separation must have a point of inflection somewhere above the wall as shown in



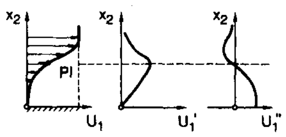
a) Full profile



b) Blasius profile



c) Inflectional profile



d) Profile at the point of steady, two-dimensional separation

Fig. 6 Normal velocity profile in a boundary layer.

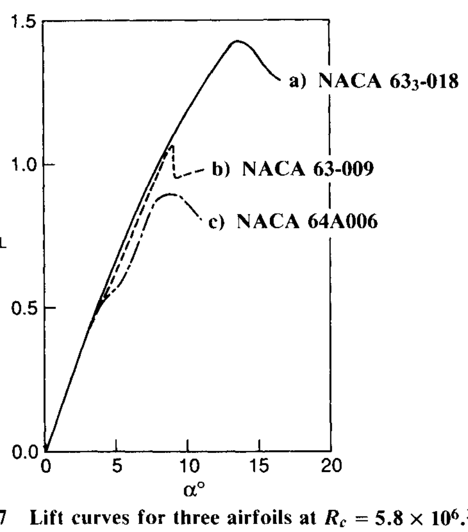
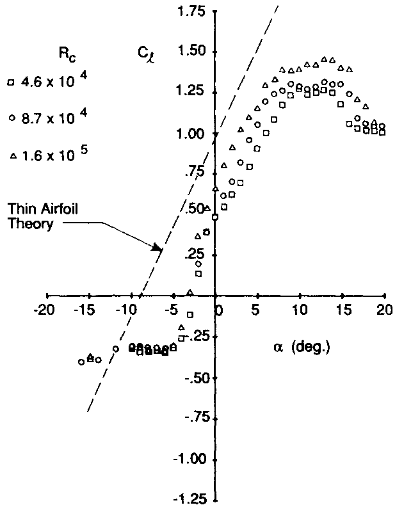
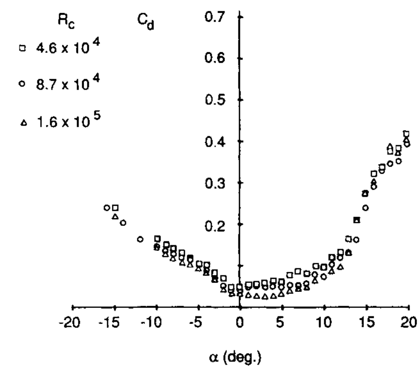


Fig. 7 Lift curves for three airfoils at $R_c = 5.8 \times 10^6$.[3]



a) Lift curves



b) Drag curves

Fig. 8 Reynolds number effects on lift and drag of the Eppler 61 airfoil.[17]

Fig. 6d. Since $[\partial^2 U/\partial y^2]_0 > 0$ is a necessary condition for a steady, two-dimensional boundary layer to separate, the opposite, i.e., a negative curvature of the velocity profile at the wall (Fig. 6a), is a sufficient condition for the boundary-layer flow to remain attached.

The above arguments naturally lead to several possible methods of control to delay separation. Namely, the object is to keep $[\partial^2 U/\partial y^2]_0$ as negative as possible, or in other words to make the velocity profile as full as possible. In this case, the mean spanwise vorticity decreases monotonically away from the wall, and the surface vorticity flux is in the positive y direction. Considering the effects of the terms in the left-hand side of Eq. (4) on the curvature of the velocity profile (or the vorticity flux) at the wall, separation control methods include the use of wall suction ($v_w < 0$), favorable pressure gradient ($dP_0/dx < 0$), surface cooling in gases ($d\mu/dT > 0$; $[\partial T/\partial y]_0 > 0$), or surface heating in liquids ($d\mu/dT < 0$; $[\partial T/\partial y]_0 < 0$). Obviously any one or a combination of these methods could be used in a particular situation. For example, beyond the point of minimum pressure on a streamlined body, the pressure gradient is adverse, and the boundary layer may separate if the pressure rise is sufficiently steep; however, enough suction could be applied there to overcome the retarding effects of the adverse pressure and to prevent separation. Each of these control methods is covered in more details in the following.

A. Separation Control

Streamlining can greatly reduce the steepness of the pressure rise. Laminar boundary layers can only support very small adverse pressure gradients without separation. As mentioned in Sec. II, if the ambient incompressible fluid decelerates in the streamwise direction faster than $U_0 \sim x^{-0.09}$, the flow separates. On the other hand, a turbulent boundary layer, being an excellent momentum conductor, is capable of overcoming much larger adverse pressure gradient without separation. In this case, separation is avoided for external flow deceleration up to $U_0 \sim x^{-0.23}$. The efficient momentum transport that characterizes turbulent flows provides the mechanism for mixing the slower fluid near the wall with the faster fluid particles further out. The forward movement of the boundary-layer fluid against pressure and viscous forces is facilitated and separation is, thus, postponed. According to the experimental results of Schubauer and Spangenberg,[31] a larger total pressure increase without separation is possible in the turbulent case by having larger adverse pressure gradient in the beginning and continuing at a progressively reduced rate of increase.

As mentioned in the introduction, transition on the upper surface of a lifting surface typically occurs at the first onset of adverse pressure gradient if the Reynolds number exceeds 10^6. The separation-resistant turbulent boundary layer that evolves in the pressure recovery region results in higher maximum lift at relatively large angle of stall. Depending on the severity of the initial adverse gradient, and hence on the airfoil shape, laminar separation may take place prior to transition. Regardless of whether or not the flow subsequently reattaches, the laminar separation leads to higher form drag and lower maximum lift. Delicate contouring of the airfoil near the minimum pressure point to lessen the severity of the adverse pressure gradient may be used to accomplish separation-free transition.

As an example of the effects of the airfoil's shape on its performance, consider the lift curves for the three sections NACA 63_3-018, NACA 63-009, and NACA 64A006. These airfoils have maximum thicknesses of 0.18 c, 0.09 c, and 0.06 c, respectively, where c is the chord. The respective leading edge radii are 0.021 c, 0.006 c, and 0.003 c. Figure 7, adapted from the measurements by McCullough and Gault,[3] depicts C_L vs α curves for the three sections at chord Reynolds number of 5.8×10^6. For the thick section, NACA 63_3-018, transition takes place near the minimum pressure point. Stalling in this case is of the trailing-edge type and is preceded by a gradual movement of the separation point of the turbulent boundary layer forward from the trailing edge as α increases. A laminar separation bubble is formed on the other two sections at small incidence. However, the NACA 63-009 section experiences a sudden leading-edge stall when the bubble bursts with no subsequent reattachment; while the NACA 64A006 section experiences a more gradual thin-airfoil stall. In the latter case, the short bubble breaks down into a longer bubble at an angle of attack of 5 deg causing a slight discontinuity in the lift curve. Subsequent increase in α leads to a movement of the reattachment point towards the trailing edge. The maximum lift in this case is about 40% lower than that for the thick airfoil. The stall angle is also lower.

A second example is provided for an airfoil specifically designed for the low-Reynolds-number regime. The Eppler 61 has a maximum thickness of 0.056 c and is highly cambered. Mueller and Burns[17] reported lift, drag, and smoke visualization data for this airfoil section in the range of Reynolds numbers of $3 \times 10^4 - 2 \times 10^5$. A sample of their lift and drag curves at three different speeds is depicted in Fig. 8. At a negative angle of attack of about -3 deg, the flow around the cambered airfoil separates, without further reattachment, at the leading edge on the lower surface. Zero lift is measured at this angle and is correlated with the appearance of smooth smokelines above and below the airfoil to form an uncambered, symmetrical shape. This explains the deviation of the zero lift angle as well as the shape of the lift curve from those predicted by the thin airfoil theory. Strong Reynolds number effects are evident in both the lift and drag curves. At increasing attack angles, the Eppler 61's performance is similar to that of the other thin airfoil depicted in Fig. 7c; although the maximum lift coefficient is higher in the former case.

The second method to avert separation involves withdrawing the near-wall fluid through slots or porous surfaces. Prandtl[30] applied suction through a spanwise slit on one side of a circular cylinder. He used (see Ref. 32) the momentum integral equation for a laminar boundary layer to make a simple estimate of the required suction:

$$C_q = 4.36\, Re^{-0.5} \tag{6}$$

where Re is the Reynolds number based on the cylinder diameter and the freestream velocity. Several researchers have used similar approximate methods to calculate the laminar boundary layer on a body of arbitrary shape with arbitrary suction distribution. A particularly simple calculation is due to Truckenbrodt.[33] He reduces the problem to solving a first-order ordinary differential equation. As an example, for a symmetrical Zhukovskii airfoil with uniform suction, Truckenbrodt predicts a suction coefficient just sufficient to prevent separation of

$$C_q = 1.12\, Re^{-0.5} \tag{7}$$

where Re is the Reynolds number based on the airfoil chord and the freestream velocity.

For turbulent boundary layers, semiempirical methods of calculation are inevitably used due to the well-known closure problem. Suction coefficients in the range of $C_q = 0.002-0.004$ are sufficient to prevent separation on a typical airfoil.[7,34] Optimally, the suction should be concentrated on the low-pressure side of the airfoil just a short distance behind the nose where, at large angles of attack, the largest local adverse pressure gradient occurs.

The third term in Eq. (4) points to yet another method to delay boundary-layer separation. By transferring heat from the wall to the fluid in liquids or from the fluid to the wall in gases, this term adds a negative contribution to the curvature of the velocity profile at the wall and, hence, causes the separation point to move farther aft. Although this method of control has been successfully applied to delay transition in both water and air flows (Sec. VI), its use to prevent separation has been demonstrated only for high-speed gaseous flows.[35-43]

B. Additional Separation Control Techniques

In his 1976 monograph, Chang[44] reviews several other passive and active methods to postpone separation for low- and high-speed flows. Common to all these control methods is an attempt to supply additional energy to the near-wall fluid particles which are being retarded in the boundary layer. Passive techniques do not require auxiliary power but do have an associated drag penalty and include intentional tripping of transition from laminar to turbulent flow upstream of what would be a laminar separation point,[45,46] boundary-layer fences to prevent separation at the tips of swept-back wings, placing an array of vortex generators on the body to raise the turbulence level and to enhance the momentum and energy in the neighborhood of the wall,[47,48] geometric design to avoid shock-induced separation, machining of a series of lateral grooves on the surface of the body upstream of separation, a rippled trailing edge,[49] streamwise corrugations,[50] or the use of a screen to divert the flow and to increase the velocity gradient at the surface. Howard and Goodman[51,52] recently investigated the effectiveness of two passive techniques to reduce flow separation: transverse rectangular grooves and longitudinal V-grooves placed in the aft shoulder region of a bluff body. Both types of grooves were beneficial in reducing the form drag on a body at zero and moderate angles of yaw.

Active methods to postpone separation require energy expenditure. Obviously, the energy gained by the effective control of separation must exceed that required by the device. In addition to suction or heat transfer reviewed earlier, fluid may be injected parallel to the surface to augment the shear-layer momentum or normal to the wall to enhance the mixing rate.[53] Either a blower is used or the pressure difference that exists on the aerodynamic body itself is utilized to discharge the fluid into the retarded region of the boundary layer. The latter method is found in nature in the thumb pinion of a pheasant, the split tail of a falcon, or the layered wing feathers of some birds. In man-made devices, passive blowing through leading-edge slots and trailing-edge flaps is commonly used on aircraft wings. Although in this case direct energy expenditure is not required, the blowing intensity is limited by the pressure differentials obtainable on the body itself.

Tangential jet blowing over the upper surface of a rounded trailing-edge airfoil sets an effective Kutta condition by fixing the location of separation. This circulation control concept was initially described by Cheeseman and Seed,[54] and a substantial data base has been gathered since then for the purpose of performance evaluation.[55-57] More recently, McLachlan[58] conducted an experimental study of the flow past a two-dimensional circulation control airfoil under steady leading/trailing-edge blowing. In the range of chord Reynolds numbers of $1.2 \times 10^5 - 3.9 \times 10^5$, McLachlan reported a large increase in the lift coefficient when trailing-edge blowing was used. When leading-edge blowing was employed simultaneously, a slight decrease in lift was observed.

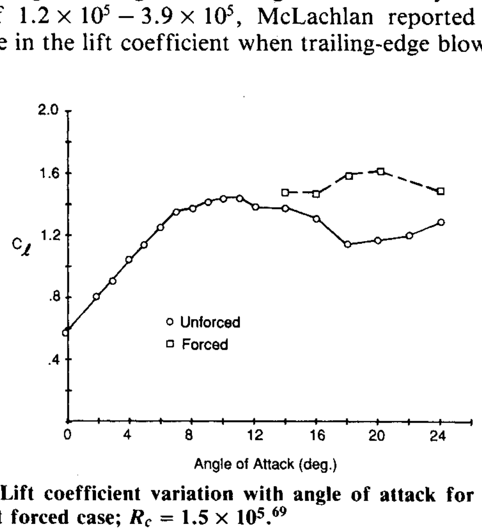


Fig. 9 Lift coefficient variation with angle of attack for unforced and best forced case; $R_c = 1.5 \times 10^5$.[69]

Other active methods for controlling boundary-layer separation and reattachment include the use of acoustic excitations,[59-63] periodic forcing of the velocity field via an oscillating flap or wire,[64-69] and oscillatory surface heating.[70] The effect of sound on the flow over airfoils at high angles of attack have been studied since 1975.[59] At chord Reynolds numbers up to 1×10^6, Ahuja and his colleagues[60,61] successfully demonstrated that sound at a preferential frequency and sufficient amplitude can postpone the turbulent separation on an airfoil in both prestall and poststall regimes. The optimum frequency was found to be $4U_\infty/c$ (Strouhal number = 4), where U_∞ is the freestream velocity and c is the airfoil chord. Goldstein[71] speculates that the delay in separation in the Ahuja et al. experiment[60] resulted from enhanced entrainment promoted by instability waves that were triggered on the separated shear layer by the acoustic excitation. Zaman et al.[62] have found that the most effective separation control is achieved at frequencies in which the acoustic standing waves in their wind tunnel induce transverse velocity fluctuations in the vicinity of an airfoil. They speculated that effective separation control can be obtained by direct introduction of velocity disturbances.

To directly disturb the velocity field, Koga et al.[64] used a computer-controlled, spoiler-like flap in a flat-plate turbulent boundary layer with and without modeled upstream separation. They were able to manipulate the separated flow region and its reattachment length characteristics by varying the frequency, amplitude, and waveform of the oscillating flap. Reynolds and Carr[72] offer a plausible explanation, from the viewpoint of a vorticity framework, for the experimental observations of Ref. 64. It seems that the large-scale vortical structures produced by forcing play a major role in enhancing mixing and entrainment, thus leading to reattachment.

Periodic forcing of the velocity field has been shown to reduce reattachment length in both laminar and turbulent flows on a number of other basic geometrical configurations.[65,67-69] At a chord Reynolds number in the range of $1 \times 10^5 - 3 \times 10^5$, Bar-Sever[69] used an oscillating wire to introduce transverse velocity fluctuations into a separated shear layer on an airfoil at high incidence. The effectiveness of this separation control technique is depicted in Fig. 9, showing the variation of unforced and forced lift coefficients as a function of angle of attack. For each angle, the forced case represents the best lift achieved at any combination of forcing frequency and amplitude. At $\alpha = 20$ deg, the controlled forcing moved the separation from the leading edge to about 0.8 c. A wide band of dimensional forcing frequencies (0.7–2.7) was found to be effective with diminishing influence at lower frequencies.

C. Three-Dimensional and Unsteady Separation

Unlike the relatively simple case of steady, two-dimensional (or axisymmetric) boundary-layer flow over a fixed wall, the point of vanishing wall shear and the occurrence of reverse flow do not necessarily indicate separation for two-dimensional flow over moving surfaces, two-dimensional unsteady flows, or three-dimensional steady flows, and a more complex criterion for separation must be sought.[73-75] For time-dependent flows, the separation point is no longer stationary but rather moves along the surface of the body. The Moore-Rott-Sears (MRS) criterion states that unsteady separation occurs when the shear and velocity vanish simultaneously and in a singular fashion at a point within the boundary layer as seen by an observer moving with the separation point.[76-78] In analogy to the moving surface case, unsteady separation is postponed when the separation point moves upstream as is the case on the suction side of an airfoil undergoing a pitching motion from small to large angles of attack. Conversely, when an airfoil is pitched from large to small attack angle, the separation point on the suction side moves downstream and separation is advanced, much the same as the case of a wall moving upstream.

Three-dimensional relief of the streamwise adverse pressure gradient could be exploited to delay separation. Properly designed corrugated trailing edges could provide sufficient easement to postpone the separation at higher angles of attack. In nature, three-dimensional serrated geometry can be found in the trailing edges of the fins and wings of many aquatic animals and birds.[79,80] For man-made lifting surfaces, the same concept was tested in the low-Reynolds-number regime by Vijgen et al.[81] They reported a modest 5% increase in the maximum lift-to-drag ratio when triangular serrations were added to the trailing edge of a natural-laminar-flow airfoil.

Although the object of Sec. V is to review control methods to prevent separation, under certain circumstances the designer may wish to provoke separation, or the flow may separate as a byproduct of another primary goal of control. Periodic separation may be provoked by changes in the surface geometry. Francis et al.[82] initiated separation on an airfoil by periodically inserting and removing a spoiler at the surface. Viets et al.[83] studied separation inducement and control by use of a cam-shaped rotor mounted on an airfoil. The cam, either driven or free-wheeling, periodically extended out into the flow causing the boundary layer to separate. Large-scale coherent spanwise structures were periodically generated and were responsible for the flow detachment.[72]

VI. Transition Control

Delaying laminar-to-turbulent transition of a boundary layer has many obvious advantages. Depending on the Reynolds number, the skin-friction drag in the laminar state can be as much as an order of magnitude less than that in the turbulent condition. On the other hand, turbulence is an efficient mixer and rates of mass, momentum, and heat transfer are much lower in the laminar state, so early transition may be sought in some applications as for example when rapid mixing or separation delay is desired.

Reshotko[84] asserts that transition is a consequence of the nonlinear response of the laminar shear layer (a very complicated oscillator) to random forcing disturbances that result from freestream turbulence, radiated sound, surface roughness, surface vibrations, or combination of these environmental factors. If the initial disturbances are small, transition Reynolds number depends upon the nature and spectrum of these disturbances, their signature, excitation of the normal modes in the boundary layer (receptivity; see Ref. 85), and the linear amplification of the growing normal modes. Once wave interaction and nonlinear processes set in, transition is quickly completed. If the initial disturbance levels are large enough, the relatively slow linear amplification step of Tollmien-Schlichting waves is *bypassed*[86] and transition can occur at much lower Reynolds numbers. In fact, a sufficiently violent disturbance, $u_{rms}/U_\infty \sim 0.1$, can cause transition of a laminar boundary layer to advance to the position upstream of which perturbations of all wave numbers decay.[87]

To delay transition to a position as far downstream as possible, the following steps may be taken. First, since factors that affect the linear amplification of Tollmien-Schlichting waves determine the magnitude of the transition Reynolds number, these waves may be either inhibited or cancelled. In the former method of control, the growth of the linear disturbance is minimized using any or a combination of the so-called stability modifiers. Wave cancellation of the growing perturbation is accomplished through exploiting but not altering the stability characteristics of the flow. Second, the forcing disturbances in the environment in which the laminar shear layer develops may be reduced. This is accomplished by using smooth surfaces, reducing the freestream turbulence and the radiated sound, minimizing body vibration, and ensuring a particulate-free incoming flow, or, in the case of a contaminated environment, using a particle-defense mechanism. Practically achieved surface smoothness and levels of radiated noise place an upper limit for unit Reynolds number required for a successful laminar flow control system. For conventional aircraft, this typically translates into a requirement for high altitude operation (above 10 km). For low-Reynolds-number aircraft, the unit Reynolds number criterion is easily met even at low altitudes. Third, one may provide a flow where other kinds of instabilities, e.g., Taylor-Görtler vortices or crossflow instabilities, will not occur or at least will not grow at a rapid rate. This is done by avoiding as much as possible concave surfaces or concave streamlines, minimizing the sweep on lifting surfaces, etc.

The Reynolds number below which perturbations of all wave numbers decay is termed the critical Reynolds number or the limit of stability. For a given velocity profile $U(y)$ the critical Reynolds number and the rate of growth of perturbations depends strongly on the shape of the velocity profile. A profile with an inflectional point ($\partial^2 U/\partial y^2 = 0$) above the wall provides a necessary and sufficient condition for inviscid instability. Such profiles must have a positive curvature at $y = 0$ since $\partial^2 U/\partial y^2$ is negative at a large distance from the wall (Fig. 6c). Even when viscous effects are included, a velocity profile becomes more stable as its second derivative near the wall becomes negative, $[\partial^2 U/\partial y^2]_0 < 0$. The profile is then said to be more full (Fig. 6a) having a smaller ratio of displacement thickness to momentum thickness than, for example, an inflectional velocity profile. In the former case, the critical Reynolds number is increased, the range of amplified frequencies is diminished, and the amplification rate of unstable waves is reduced.

Stability modifiers are those methods of laminar flow control which alter the shape of the velocity profile to minimize the linear growth of unstable waves. For a two-dimensional laminar boundary layer, vorticity is only in the spanwise direction and is given by $\Omega z \approx -\partial U/\partial y$. Any of the terms on the left-hand side of Eq. (4) can affect the sign of the second derivative of the velocity profile (or the direction of the vorticity flux) at the wall and, hence, the flow stability. Boundary layers which are stabilized by extending the region of favorable pressure gradient are known as *natural laminar flow* (NLF); while the other methods to modify the stability of the shear flow are termed *laminar flow control* (LFC). It is clear from Eq. (4) that the effects of these methods are additive. The term hybrid laminar flow control normally refers to the combination of NLF and one of the LFC techniques.

A. Suction

We first consider the active control of transition using wall suction. Small amounts of fluid withdrawn from the surface can greatly alter the stability characteristics of the boundary layer. Additionally, suction inhibits the growth of the boundary layer so that the critical Reynolds number based on thickness may never be reached. Although laminar flow can be maintained to an extremely high Reynolds number provided that enough fluid is sucked away, the goal is to accomplish transition delay with the minimum suction flow rate. This will minimize the momentum loss due to suction and, hence, the skin friction. This point can easily be seen from the momentum integral equation. Rewriting Eq. (5) for a flat plate ($dU_0/dx = 0$; $dR/dx = 0$) with uniform suction through the wall (v_w negative), the equation reads

$$\frac{C_f}{2} = \frac{d\delta_\theta}{dx} + \frac{|v_w|}{U_0} \tag{8}$$

The second term on the right-hand side is the suction coefficient C_q and although withdrawing the fluid through the wall leads to a decrease in the rate of growth of the momentum thickness, C_f increases directly with C_q. Fluid withdrawn through the surface has to come from outside the boundary layer where the streamwise momentum per unit mass is at the relatively high level of U_0. The second term is proportional to the rate of momentum loss due to withdrawing a mass per unit time and area of $\rho|v_w|$.

Delaying transition using suction is a mature technology, where most of the remaining problems are in the maintainability and reliability of suction surfaces and the optimization of suction rate and distribution. To protect the delicate suction surfaces on the wing of an aircraft from insect impacts and ice formation at low altitudes, special leading edge systems are used.[88,89]

B. Shaping to Control Transition

A second method of control to delay laminar-to-turbulent transition is a passive one and involves the use of suitably shaped bodies to manipulate the pressure distribution. In Eq. (4), the pressure gradient term can affect the sign of the curvature of the velocity profile at the wall and, hence, change the stability characteristics of the boundary layer. The critical Reynolds number based on displacement thickness and freestream velocity changes from about 100 to 10,000 as a suitably nondimensionalized pressure gradient (the shape factor, Λ) varies from $\Lambda = -6$ (adverse) to $\Lambda = +6$ (favorable). Moreover, for the case of a favorable pressure gradient, no unstable waves exist at infinite Reynolds number. In contrast, the upper branch of the neutral stability curve in the case of an adverse pressure distribution tends to a nonzero asymptote so that a finite region of wavelengths at which disturbances are always amplified remains even as $Re \rightarrow \infty$.

Streamlining a body to prevent separation and to reduce form drag is quite an old art, but the stabilization of a boundary layer by pushing the longitudinal location of the pressure minimum to as far back as possible dates back to the 1930s and led to the successful development of the NACA 6-Series NLF airfoils. Newer, low-Reynolds-number lifting surfaces used in sailplanes, low-speed drones, and executive business jets have their maximum thickness point far aft of the leading edge. The recent success of the Voyager's nine-day, unrefueled flight around the world was due in part to a wing design employing natural laminar flow to approximately 0.5 c.

The favorable pressure gradient extends to the longitudinal location of the pressure minimum. Beyond this point, the adverse pressure gradient becomes steeper and steeper as the peak suction is moved further aft. For an airfoil, the desired shift in the point of minimum pressure can only be attained in a certain narrow range of angles of incidence. Depending on the shape, angle of attack, Reynolds number, surface roughness, and other factors, the boundary layer either becomes turbulent shortly after the point of minimum pressure or separates first and then undergoes transition. One of the design goals of NLF is to maintain attached flow in the adverse pressure gradient region and some method of separation control (Sec. V) may have to be used there.

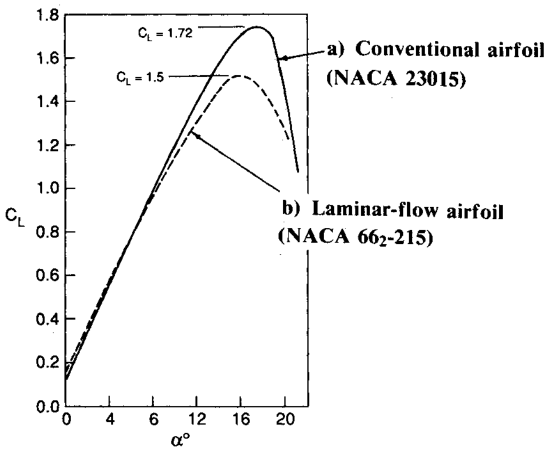


Fig. 10 Lift curves for a conventional section and a laminar-flow section at $R_c = 9 \times 10^6$.[90]

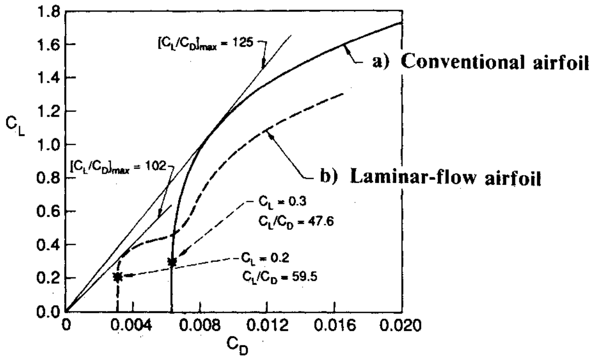


Fig. 11 Lift-drag polars for the two airfoils depicted in Fig. 10.[90]

Subtle changes in the magnitude and extent of favorable pressure gradient, leading-edge radius, and other shape variables can have pronounced effects on the airfoil's performance. As an example, consider the lift and drag characteristics of a conventional section (NACA 23015) and a laminar-flow section (NACA 66_2-215). Both airfoils are cambered and both have maximum thickness of 0.15 c. However, the maximum thickness point is located at 0.3 c for the conventional section and at 0.45 c for the laminar-flow one. The leading edge radius is 0.025 c and 0.014 c for the two respective airfoils. The point of minimum pressure, as computed for the basic symmetric section at zero incidence, is located at 0.15 c for the conventional airfoil; while this point is pushed back to 0.6 c for the laminar-flow airfoil. As seen from the lift curves depicted in Fig. 10 for chord Reynolds number of 9×10^6, the laminar-flow section has slightly higher lift at small angles of attack than the conventional section, but stalling occurs at lower incidence and the maximum lift is smaller for the laminar-flow airfoil. The lift-drag polars for the same two airfoils are shown in Fig. 11. The sudden increase in drag for the laminar-flow section occurs at an angle of attack of about 1.5 deg. This is caused by a forward movement of the minimum pressure point and corresponding accentuation of the adverse pressure gradient as the flow on the upper surface accelerates sharply to round the airfoil's sharp nose. The increased adverse pressure gradient leads to early transition and corresponding drag increase. The maximum lift-to-drag ratios for the conventional and laminar-flow airfoils are 125 and 102, respectively. However, the laminar-flow section is designed to cruise in the low-drag region ($\alpha < 1.5$ deg). At the design lift coefficients of 0.3 and 0.2 for the conventional and laminar-flow sections, the lift-to-drag ratios are 47.6 and 59.5, respectively.

Factors that limit the utility of NLF include crossflow instabilities and leading-edge contamination on swept wings, insect and other particulate debris, high unit Reynolds numbers of conventional aircraft at lower cruise altitudes (not a limiting factor for low-Reynolds-number aircraft), and performance degradation at higher angles of attack due to the necessarily small leading edge radius of NLF airfoils. Reductions of surface waviness and smoothness of modern production wings, special leading-edge systems to prevent insect impacts and ice accretion,[91-93] higher cruise altitudes of newer airplanes, and higher Mach numbers all favor the application of NLF.[94]

C. Control of Transition via Heating/Cooling

The third stability modifier is an active one and involves the addition or removal of heat from a surface, which causes the viscosity to vary with distance from the wall. In general, viscosity increases with temperature for gases; while the opposite is true for liquids. Thus, if heat is removed from the surface of a body moving in air, the third term on the left-hand side of Eq. (4) is negative. In that case, the velocity gradient near the wall increases and the velocity profile becomes fuller and more stable. The term containing the viscosity derivative will also be negative if the surface of a body moving in water is heated. With heating in water or cooling in air, the critical Reynolds number is increased, the range of amplified frequencies is diminished, and the amplification rate of unstable waves is reduced. Substantial delay of transition is feasible with a surface that is only a few degrees hotter (in water) or colder (in air) than the freestream. For aircraft, this method of transition delay is feasible only for a vehicle which uses a cryo-fuel such as liquid hydrogen or liquid methane. In this case, a sizeable heat sink is readily available: the fuel is used to cool the major aerodynamic surfaces of the aircraft as it flows from the fuel tanks to the engines.

D. Wave Superposition to Control Transition

An alternative approach to increase the transition Reynolds number of a laminar boundary layer is wave cancellation. If the frequency, orientation, and phase angle of the dominant element of the spectrum of growing linear disturbances in the boundary layer is detected, a control system and appropriately located disturbance generators may then be used to effect a desired cancellation or suppression of the detected disturbances. In this case, the stability characteristics of the boundary layer are exploited but not altered.[95] Wave cancellation is feasible only when the disturbances are still relatively small, their growth is governed by a linear equation, and the principle of superposition is still valid.

The first reported use of wave cancellation is that due to Schilz.[96] He used a vibrating ribbon to excite a T-S wave on a test plate which had a flexible surface. A unique wall-motion device flush mounted into the plate moved the flexible wall in a transverse, wave-like manner with a variety of frequencies and phase speeds. A significant amount of cancellation resulted when the flexible wall motion had the opposite phase but the same frequency and phase speed as the T-S wave. Both Milling[97] and Thomas[98] used two vibrating wires, one downstream of the other, to generate and later cancel a single frequency T-S wave. Thomas[98] observed that interaction between the primary disturbance and background excitations prevented complete cancellation of the primary wave. To further study the consequences of wave interactions, Thomas applied the same method of control to eliminate two interacting waves of different frequency. Although the primary waves were behaving linearly, a nonlinear interaction gave rise to a low-amplitude difference frequency that could only be partially reduced and ultimately led to transition. Thomas[98] concluded that it is not possible to return the flow completely to its undisturbed base state because of wave interactions and that it is perhaps more appropriate to describe this control method as wave superposition rather than wave cancellation.

The same principle of wave superposition could be applied using wall heating/cooling,[99-101] plate vibration,[102] compliant wall,[103-105] or periodic suction/injection.[106] The transition delay achieved by active wave cancellation is modest, typically a factor of two or less increase in the transition Reynolds number based on distance from the leading edge. Reshotko[95] maintains that to achieve significant delay in transition using this technique would require an extensive array of disturbance detectors and generators as well as a prohibitively complicated control system that could cancel both the primary and residual disturbance spectra.

E. Turbulators

A turbulent boundary layer is more resistant to separation than a laminar one, and transition advancement may be desired in some situations. In low-Reynolds-number terminology, the transition promoting devices are called *turbulators.* For a zero-pressure-gradient boundary layer, transition typically occurs at a Reynolds number based on distance from the leading edge of the order of 10^6. The critical *Re* below which perturbations of all wave numbers decay is about 6×10^4. To advance the transition Reynolds number, one may attempt to lower the critical *Re,* to increase the growth rate of Tollmien-Schlichting waves, or to introduce large disturbances that can cause *bypass* transition. The first two routes involve altering the shape of the velocity profile using wall motion, injection, adverse pressure gradient, or surface heating in gases or cooling in liquids. The third route is much simpler to implement though more difficult to analyze.[107-110] Morkovin[111] broadly classifies the large disturbances that can cause bypass transition into steady or unsteady ones originating into the freestream or at the body surface. The most common example is single, multiple, or distributed roughness elements placed on the wall. The mechanical roughness elements, in the form of serrations, strips, bumps or ridges, are typically placed near the airfoil's leading edge. If the roughness characteristic length is large enough, the disturbance introduced is nonlinear and bypass transition takes place. For a three-dimensional roughness element of height-to-width ratio of one, a transition Reynolds number $R_{\delta^*} \simeq 300$ (below the critical $R_{\delta^*} = 420$ predicted from the linear stability theory) is observed for a roughness Reynolds number, based on its height and the velocity in the undisturbed boundary layer at the height of the element, of about 10^3.[112] Transition occurs at $R_{\delta^*} \simeq 10^3$ for a roughness Reynolds number of about 600. For a smooth surface, transition typically takes place at $R_{\delta^*} \simeq 2.6 \times 10^3$. An important consideration when designing a turbulator is to produce turbulence and suppress laminar separation without causing the boundary layer to become unnecessarily thick. A thick turbulent wall-bounded flow suffers more drag and is more susceptible to separation than a thin one. Consistent with this observation, available data (see Fig. 1) indicate that a rough airfoil has higher lift-to-drag ratio than a smooth one for $R_c < 10^5$, but that this trait is reversed at higher Reynolds numbers.

For low-Reynolds-number airfoils, performance may be improved by reducing the size of the laminar separation bubble through the use of *transition ramps*[113] or boundary-layer trips.[114-116] Donovan and Selig[116] provide extensive data using both methods for 40 airfoils in the Reynolds number range of $6 \times 10^4 - 3 \times 10^5$. A long region of roughly constant adverse pressure gradient on the upper surface of a lifting surface (termed a bubble ramp) achieved a lower drag than the more conventional laminar-type velocity distribution in which initially the pressure remains approximately constant and then quickly recovers. Trips were also used to shorten the separation bubble. A simple, two-dimensional trip performed as well or better than zig-zag tape, hemisphere bumps, and normal blowing. Donovan and Selig concluded that an airfoil which performs poorly at low Reynolds numbers can be improved through the use of turbulators. Trips were less effective, however, at improving airfoils which normally had low drag.

Other large disturbances that could lead to early transition include high turbulence levels in the freestream, external acoustic excitations, particulate contamination, and surface vibration. These are often termed "environmental tripping." Transition could also be effected by detecting naturally occurring T-S waves and artificially introducing in-phase waves. Transition could be considerably advanced, on demand, using this wave superposition principle.

Early transition could also be achieved by exploiting other routes to turbulence such as Taylor-Görtler or crossflow vortices. For example, a very mild negative curvature of $(0.003/\delta^*)$ results in the generation of strong streamwise vortices. In this case, transition Reynolds number is lowered from $R_{\delta^*} \simeq 2600$ for the flat-plate case to $R_{\delta^*} \simeq 700$ for the curved surface.[112] For high Mach number flows, the general decay in spatial amplification rate of T-S waves makes conventional tripping more difficult as the Mach number increases.[117] For these flows, trips that generate oblique vorticity waves of appropriate wavelength may be most effective to advance the transition location.

The last issue to be considered in this section is how to augment the turbulence for a shear flow that has already undergone transition. The newly developed turbulent flow is less capable of resisting separation than a corresponding flow at higher Reynolds numbers. Turbulence augmentation in the low-Reynolds-number case is then a useful control goal to energize the flow and to enhance its ability to resist separation at higher angles of attack. Roughness will enhance the turbulence, but its associated drag must be carefully considered. Other devices to enhance the turbulent mixing include vane-type vortex generators, which draw energy from the external flow, or wheeler-type or Keuthe-type generators, which are fully submerged within the boundary layer and presumably have less associated drag penalty.[48]

VII. Drag Reduction

A. Laminar Regime

The total drag experienced by a lifting surface consists of skin friction and pressure drag. Flow separation is the major source of pressure drag with additional contributions due to displacement effects of the boundary layer, wave resistance in a supersonic flow or at an air/water interface, and drag induced by lift on a finite body. For a vehicle, the reduced drag means longer range, reduced fuel cost/volume, higher payload, or increased speed; all potentially important considerations when designing an RPV for military or scientific applications.

Streamlining and other control methods summarized in Sec. V can eliminate most of the pressure drag due to flow separation. Due to the displacement effects of the boundary layer, some form drag remains even when the flow remains attached to the trailing edge. This remanent drag can be reduced by keeping the boundary layer as thin as possible. Wave resistance and induced drag can also be reduced by geometric design.

In the absence of transition promoters, such as crossflow, concave surfaces, adverse pressure gradient, roughness, or freestream disturbances, the boundary layer is laminar to $Re = 0$ [10^6]. In this case, methods to reduce the laminar skin-friction are sought. This may be useful for some land vehicles, airborne vehicles at very high altitude, small RPVs, or small hang gliders and the like.

From Eq. (4), it is clear that any or a combination of the following techniques can be used to lower C_f: injection of fluid normal to the wall; adverse pressure gradient; wall heating in air; or wall cooling in water. Note that any of these methods will promote flow instability (see Sec. VI) and separation (see Sec. V). These tendencies have to be carefully considered when deciding how far to go with the attempt to lower C_f. Moreover, energy expenditure for the active techniques must be less than the energy saved due to the lower skin friction.

Two other techniques can be used to lower laminar skin friction. Narasimha and Ojha[118] considered the higher-order effects of moderate longitudinal surface curvature. Their similarity solutions show a definite decrease in skin friction when the surface has convex curvature in all cases including zero pressure gradient. Narasimha and Ojha attributed the decrease in C_f to the fact that the velocity in the potential flow region tends to decrease away from the surface. The second technique is used in rarefied gas flows. Appropriate surface preparation could be used to introduce a slip velocity at the wall and, thus, lower the tangential momentum accommodation coefficient.[119-120]

B. Turbulent Regime

A design goal for low-Reynolds-number airfoils is to reduce the skin friction in the turbulent region downstream of the separation bubble while maintaining attached flow as far aft as possible. Techniques to reduce the turbulent skin-friction coefficient are classified in four categories: reduction of near-wall momentum; introduction of foreign substances; geometrical modifications; and synergism. Methods that rely on reducing the near-wall momentum are similar to those used in the laminar case. The influence of wall transpiration, shaping, or heat transfer on the mean velocity profile is complicated by the additional effects of these modulations on the Reynolds stress term. However, these influences are qualitatively in the same direction as in the simpler laminar case. Thus, lower skin friction is achieved by driving the turbulent boundary layer towards separation. This is accomplished by injecting fluid normal to the wall, shaping to produce adverse pressure gradient, surface heating in air, or surface cooling in water.

Although in the reverse flow region downstream of the separation line, the skin friction is negative, the increase in pressure drag is far more than the saving in skin-friction drag. The goal of the above methods of control is to avoid actual separation, i.e., lower C_f but not any lower than zero. The papers by Stratford[121,122] provide useful discussion on the prediction of turbulent-boundary-layer separation and the concept of flow with continuously zero skin-friction throughout its region of pressure rise. By specifying that the turbulent boundary layer be just at the condition of separation, without actually separating, at all positions in the pressure rise region, Stratford[122] experimentally verified that such a flow achieves a specified pressure rise in the shortest possible distance and with the least possible dissipation of energy. An airfoil which could utilize the Stratford's distribution immediately after transition from laminar to turbulent flow would be expected to have a very low drag. Liebeck[123] successfully followed this strategy using a highly polished wing to achieve the best lift-to-drag ratio (over 200) of any airfoil tested in the range of Reynolds numbers of $5 \times 10^5 - 2 \times 10^6$. He argued that the entire pressure-recovery region of an airfoil's upper surface would be operating at its maximum capacity if the adverse pressure distribution was uniformly critically close to separation. By assuming an incipient-separation turbulent profile, Liebeck calculated the pressure field required then used an inverse calculation procedure to derive the airfoil shape from the given critical-velocity distribution.

When attempting to reduce drag by driving the boundary layer towards separation, a major concern is the flow behavior at off-design conditions. A slight increase in angle of attack for example can lead to separation and consequent large drag increase as well as loss of lift. High performance airfoils with lift-to-drag ratio of over 100 utilize carefully controlled adverse pressure gradient to retard the near-wall fluid, but their performance deteriorates rapidly outside a narrow envelope.[124]

Turbulent skin-friction drag can also be reduced by the addition of several foreign substances. Examples include long-chain molecules, surface-active agents, and microbubbles in liquid flows, and small solid particles or fibers in either gases or liquids. In general, the addition of these substances leads to a suppression of the Reynolds stress production in the buffer zone that links the linear and the logarithmic portions of the mean velocity profiles. The turbulent mixing is thus inhibited, and a consequent reduction in the viscous shear stress at the wall is achieved. Numerous review and technical articles of drag reduction methods involving the introduction of a foreign substance are available.[28,125-128]

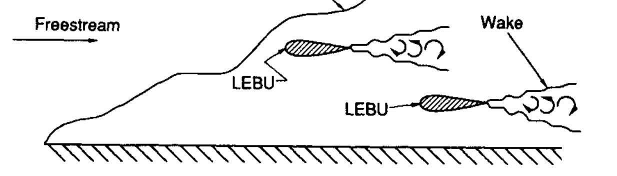


Fig. 12 Sketch of a large Eddy breakup device in a turbulent boundary layer.

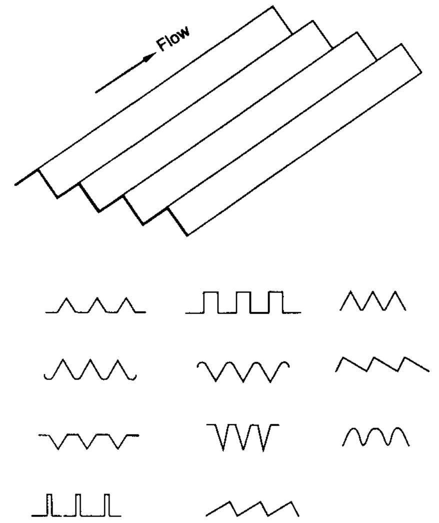


Fig. 13 Fin shapes for longitudinally ribbed surface.

The most recently researched techniques to reduce the turbulent skin-friction drag involve geometric modifications. These include large eddy breakup devices,[129,130] riblets,[131,132] compliant surfaces,[133,134] wavy walls,[135,136] and other surface modifications. The large eddy breakup devices (LEBUs) are designed to sever, alter, or break up the large vortices that form the convoluted outer edge of a turbulent boundary layer. A typical arrangement consists of one or more splitter plates placed in tandem in the outer part of a turbulent boundary layer, as sketched in Fig. 12. In low-Reynolds-number experiments, very thin elements placed parallel to a flat plate have a device's total drag that is nearly equal to laminar skin friction. A net drag reduction of the order of 20% is feasible with two elements placed in tandem with a spacing of $0[10\delta]$.[129,130] These ribbons have typically a thickness and a chord of the order of 0.01δ and δ, respectively, and are placed at a distance from the wall of 0.8δ.

Several experiments using LEBUs report a more modest drag reduction[137] or even a drag increase, but it is believed that a slight angle of attack of the thin element can result in a laminar separation bubble and a consequent increase in the device pressure drag. Flat ribbons at a small, positive angle of attack produce larger skin-friction reductions. This is consistent with the analytical result that a device producing positive lift away from the wall is more effective.[138] In any case, a net drag reduction may be achievable, at least for devices having chord Reynolds numbers $< 10^6$, if extra care is taken to polish and install the LEBU. This device's Reynolds number requirement is, of course, easily met for low-Reynolds-number aircraft providing a strong incentive for using LEBU devices on RPVs and the like. Net drag reduction with LEBUs has been documented even in the presence of favorable or adverse pressure gradient in the main flow.[139,140] Additionally, Anders and Watson[141] have demonstrated that an LEBU having an airfoil shape is nearly as effective in reducing the net drag as a flat ribbon. A wing is several orders of magnitude stiffer than a thin ribbon, a certain advantage for extending the technique to field conditions.

The second geometrical modification is the riblets. Small longitudinal striations in the surface, interacting favorably with the near-wall structures in a turbulent boundary layer, can produce a modest drag reduction in spite of the increase in surface area. The early work employed rectangular fins with height and spacing of $0[100\nu/u^*]$. The turbulent bursting rate was reduced by about 20%, and a modest 4% net drag reduction was observed.[142] In a later refinement of this technique, Walsh and his colleagues at NASA-Langley examined the drag characteristics of longitudinally ribbed surfaces having a wide variety of fin shapes (see Fig. 13) that included rectangular grooves, V-grooves, razor blade grooves, semicircular grooves, and alternating transverse curvature.[143-145] A net drag reduction of 8% is obtained with V-groove geometry with sharp peak and either sharp or rounded valley. Optimum height and spacing of the symmetric grooves are about $15\nu/u^*$. For the typical low-Reynolds-number aircraft, the required height and spacing of the grooves exceeds 350 μm. Thin, low-specific-gravity plastic films with the correct geometry on one side and an adhesive on the other side are presently available commercially, and existing vehicles could be readily retrofitted. In water, riblets were employed on the rowing shell during the 1984 Summer Olympic by the United States rowing team. Similar riblets were also used on the submerged hull of the winner of the 1987 America's Cup yacht race, the Stars and Stripes, with apparent success.

Riblets are effective in the presence of moderate adverse and favorable pressure gradients. The loss of drag-reducing effectiveness as flow conditions vary off design is gradual. Moreover, the percentage drag reduction slowly decreases to zero as the yaw angle between the flow and the grooves goes up to 30 deg. Surprisingly, drag does not increase until yaw angles >30 deg. Remaining practical problems include cost, weight penalty, particulate clogging, ultraviolet radiation, film porosity, and resistance to hydraulic fluids and fuel.

In addition to the drag reduction techniques surveyed in here, many others are available. The interested reader is referred to the review articles by Bushnell,[146] Bandyopadhyay,[147] Wilkinson et al.,[148] Bushnell and McGinley,[149] Blackwelder,[150] and Gad-el-Hak.[28] In particular, the last article on this list presents an extensive discussion of the value of combining more than one drag reduction technique to achieve a synergetic effect.

VIII. Summary

Available and contemplated flow control methods particularly suited for low-Reynolds-number lifting surfaces have been surveyed. The flow around these airfoils is dominantly affected by the formation of a separation bubble. The laminar separation makes the interrelation among transition, separation, lift and drag controls particularly salient presenting an additional degree of complexity. Several of these control techniques are suited for high- as well as low-speed airfoils.

Low-Reynolds-numbers cover the range of 10^4–10^6. In this regime, laminar separation, transition, and reattachment may all occur within a short distance on the upper surface of an airfoil at incidence. The precise conditions for the occurrence of a laminar separation bubble depend on the local Reynolds number, pressure distribution, surface curvature, surface roughness, and freestream turbulence as well as other environmental factors.

To control the flow around a low-Reynolds-number airfoil to achieve improved performance, one must carefully consider potential conflicts in trying to achieve a particular control goal while inadvertently causing an adverse effect on another goal. A laminar boundary layer is less able to resist separation but is characterized by a very low skin-friction drag. However, if the flow separates, lift decreases and form drag increases substantially. In the low-Reynolds-number regime, laminar separation is often proceeded by transition to turbulence in the separated free-shear layer and subsequent reattachment to form a closed bubble. Bubble bursting at higher incidence leads to loss of lift and increased drag.

The effect of many of the control methods reviewed in this article is explained in terms of the behavior of the spanwise vorticity flux at the wall. The fullness of the normal velocity profile is related to the direction of this vorticity flux, which in turn has a direct influence on the stability, separation, skin friction, and turbulence levels.

Stability modifiers inhibit the linear amplification of Tollmien-Schlichting waves and, therefore, determine the magnitude of the transition Reynolds number. Shaping to provide extended regions of favorable pressure gradient is the simplest method of control and is well suited for the wings of low- or moderate-speed aircraft. Moderate surface cooling in air or heating in water also increases the transition *Re* by an order of magnitude, but a sink/source of heat must be available to achieve net drag reduction. For futuristic aircraft using cryo-fuel, surface cooling may be a feasible method to delay transition.

The above stability modifiers change the shape of the velocity profile making it more full. Therefore, two-dimensional, steady separation can be postponed using the same techniques. Other separation control methods include passive ones, such as intentional tripping, fences, or vortex generators, and active devices, such as tangential injection, acoustic excitations, or oscillating surface flaps. A drag penalty is associated with both passive and active devices.

Techniques to reduce the pressure drag are more well established than turbulent skin-friction reduction techniques. Streamlining and other methods to postpone separation can eliminate most of the form drag. The wave and induced drag contributions to the pressure drag can also be reduced by geometric design. For the purpose of reducing the skin friction, three flow regimes are identified. First, for low Reynolds numbers, the flow is generally laminar, and skin friction may be lowered by reducing the near-wall momentum. Adverse pressure gradient, blowing and surface heating/cooling could lower the skin friction but increase the risk of transition and separation. Second, for moderate speeds, active and passive methods to delay transition could be used, thus, avoiding the much higher turbulent skin friction. Third, for the turbulent portion of the boundary layer downstream of the separation bubble, methods to reduce the large mean vorticity at the wall, $[\bar{\Omega}_z]_0$, are possible. These methods are classified in the following categories: reduction of near-wall momentum; introduction of foreign substances; geometrical modifications, and synergism.

Recently introduced techniques mostly fall under the third category above and seem to offer more modest net drag reduction. These methods are, however, still in the research stage and include riblets ($\sim 8\%$), large eddy breakup devices ($\sim 20\%$), and convex surfaces ($\sim 20\%$). Potential improvement in these and other methods will perhaps involve combining more than one technique aiming at achieving a favorable effect that is greater than the sum.

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